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Seismic Experiments on the North German Explosions, 1946 to 1947

P. L. Willmore

Phil. Trans. R. Soc. Lond. A 1949 **242**, 123-151
doi: 10.1098/rsta.1949.0007

SEISMIC EXPERIMENTS ON THE NORTH GERMAN EXPLOSIONS, 1946 TO 1947

By P. L. WILLMORE, *Department of Geodesy and Geophysics, Cambridge*

(*Communicated by E. C. Bullard, F.R.S.—Received* 23 *September* 1948)

CONTENTS

Seismic waves produced by explosions near Soltau were observed at distances up to 50 km., and others from the Heligoland explosion from 50 to 1000 km. Special time signals and a high recording speed enabled the instant of a sharp onset to be determined to 0·1 sec. Short-range seismic data were used to eliminate some of the effect of rocks near the surface.

The average velocity of the first arrivals was 4·4 km./sec. between 4 and 24 km. from the shot point, 5·95 km./sec. between 24 and 120 km., and 8·18 km./sec. beyond 120 km. Significant local variations were found at the shorter distances. Alternative hypotheses covering the distribution of velocity in the upper layers gave estimates of 27·4 and 29·6 km. for the depth of the ultrabasic layer. Later arrivals proved difficult to identify, and a statistical method was used to estimate the significance of travel-time curves drawn through selected groups of onsets. This test showed that P^* was not significantly recorded, but a number of onsets at 7 or 8 sec. after P_n probably represented a wave travelling for most of its path in the ultrabasic layer and reflected at the critical angle between that layer and the surface. The test failed to decide whether the onsets close to the expected times of P_g should be treated as one or more phases. Confused motion persisted during the period when transverse waves were expected, but, with the possible exception of S_n, there was no significant concentration of observations about lines representing recognized phases.

The thermal energy of the Heligoland explosion was $1·3 \times 10^{20}$ ergs, and the energy in the seismic waves was of the order of 10^{17} ergs. The efficiency was therefore comparable with that of a surface explosion, and measurements of the crater confirmed that the rock which covered the charge could not have had much effect on the momentum entering the ground.

1. Introduction

The existence of large stores of ammunition at the end of the war had given rise to the suggestion that controlled explosions could be used as a source of seismic waves, with an intensity comparable with those produced by small earthquakes. A detailed consideration of the possibilities was undertaken by a joint committee of the Royal Society and the Ministry of Supply early in 1946. It proved impracticable to arrange the experiments in England, as explosions of the necessary size on the available sites might have caused damage to property. In July 1946, however, the Explosives Storage and Transport Committee of the War Office began a series of experiments on ammunition dumps near Soltau in north Germany and readily agreed to assist in the extra arrangements required for seismic work. In the following spring, the Royal Navy blew up the stores and fortifications on Heligoland, and thereby provided material for experiments over a much greater range.

The present author was sent out under the auspices of the Royal Society to co-ordinate the operations of the various parties. Generous facilities were provided by the Control

Commission and by many other institutions in Germany. The organization of the experiments was described in a preliminary report in *Nature* (Willmore 1947), and the results were discussed at a meeting of the Royal Society on 20 November 1947.

2. Organization

Ten charges were fired in the Soltau experiments, each involving up to 40 tons of explosive. The tremors were recorded at distances up to 50 km. by nine parties from German geophysical institutions. Time marks were put on the records by clocks at the field stations, and were synchronized by special broadcast time signals. The instants of detonation were determined from seismograms obtained within 4 km. of the shot point, using the known travel times for short distances.

The Heligoland explosion took place on 18 April 1947. On this occasion the instant of detonation was determined by means of two radio transmitters on the island, whose output was received and recorded by the listening station at Altenwalde. The transmitters were seated on explosive charges which were connected by lengths of instantaneous fuse to the main firing circuit, but they continued to give some output for several seconds after the appearance of violent fluctuations on the record. The time of the final cessation of output is inconsistent with the seismic data and with the time at which the firing key was pressed. It has therefore been assumed that the small charges misfired, and that the fluctuations show the instant at which the transmitters were struck by the blast of the main explosion.

The charge contained nearly 4000 tons of high explosive, all lying within about 1 km. of the centre. About two-thirds of the total was contained within a radius of 100 m., and it is probable that the spread in the position of the charge was no more important than that of the firing time, which apparently occupied a few tenths of a second. The sharpness of the onsets at the nearest stations indicates that the main part of the shock occurred within a few hundredths of a second, and therefore suggests that the most conspicuous part of the disturbance came from the central group. The position of the epicentre is taken as lat. $54^\circ 10' 49''$ N, long. $7^\circ 53' 10''$ E, and the instant of detonation as 10 hr. 59 min. 58·5 sec. G.M.T.

Recordings were made along three profiles in Germany, one running south through the province of Oldenburg, one to the south-east, terminating at Göttingen, and one running across Schleswig Holstein. A fourth line of field stations was operated in Denmark and a six-channel seismometer was placed near Spijk, just inside the Dutch border. The shock was also recorded by thirteen permanent observatories at distances ranging up to 998 km. (Puy de Dôme) and by H.M.S. *Nepal* and the German trawler *Astrid* in the North Sea. The positions of the Heligoland stations are shown on figure 1 *a*, whilst those in the neighbourhood of Soltau are shown on a larger scale in figure 1 *b*. The American stations referred to in *Nature* are not included, as their records have not been made available to the author.

The arrival times and the instant of detonation were referred to a series of radio time signals which were specially transmitted by the B.B.C. European service. These signals, which consisted of dots at 1 sec. intervals, were recorded by nearly all the observing stations. The distance of each station was determined by computing the quantity $\sqrt{[2(1-\cos\delta)]}$ from the geocentric co-ordinates of the shot point and the station and multiplying by the

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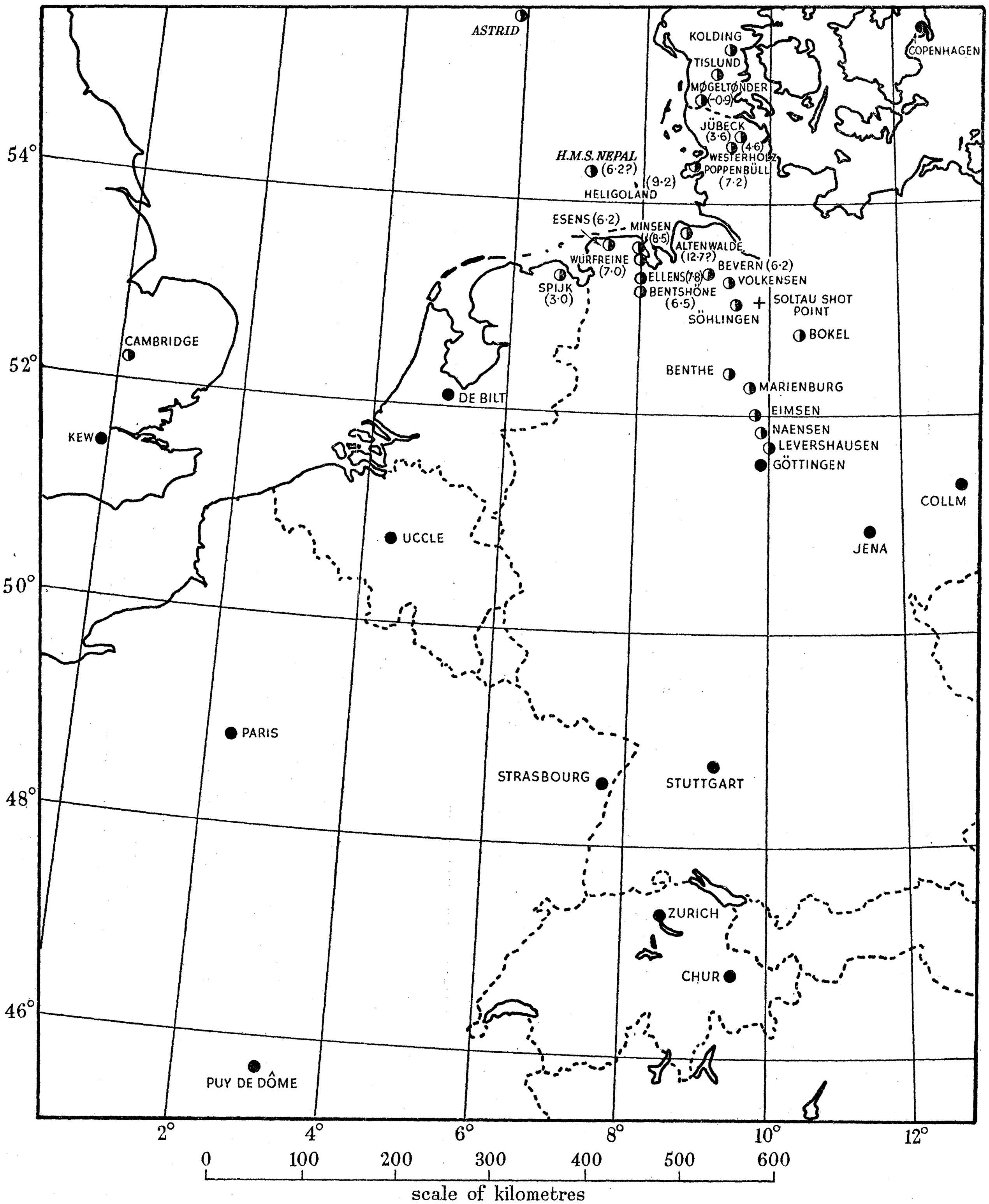

FIGURE 1*a*. Stations recording the Heligoland explosion. Figures in brackets indicate the thicknesses of the upper layer which would produce the observed travel times, if the velocities of P_s and P_g were 4·4 and 5·95 km./sec. respectively. Symbols: ● Permanent observatories, ◑ Field stations.

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radius vector of the spheroid half-way between the two. The angle δ is the one subtended at the centre of the earth, by the station and the shot point. The symbol Δ will be used below to represent the distance in km. along the chord.

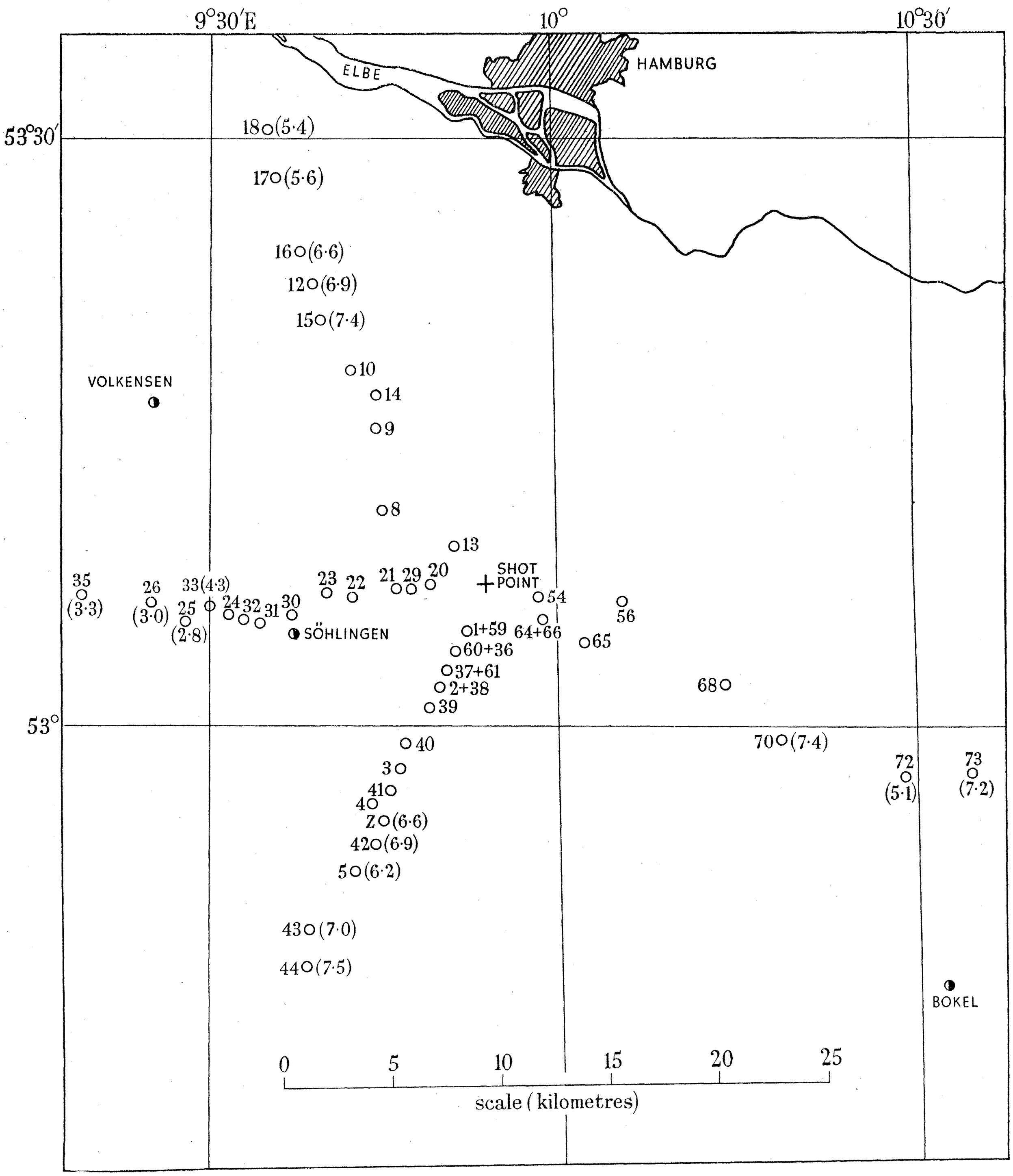

FIGURE 1*b*. Stations near Soltau. The numbers correspond with table 1 and with the preliminary German report. Figures in brackets indicate the thicknesses of the upper layer. Symbols: ◑ Heligoland stations, ○ Soltau stations.

3. Treatment of results

(a) *Characteristics of records*

The radio signals provided a time scale which was far more consistent than the experiment required, so that the uncertainty in the time of a given onset was determined almost entirely by the accuracy of reading the record. At most of the field stations the recording speed exceeded 1 cm./sec., and a sharp arrival could often be read with confidence to within 0·1 sec. The standard deviation for a single observation of P_n was 0·19 sec., which is much less than the value of 2 sec. which has been given for earthquake studies (Jeffreys 1947 *b*).

The high accuracy of reading the records suggested that most of the residuals would be due to variations of geological structure, and made it unnecessary to weight the results of the different instruments according to the quality of the recording system. A distinction between strong and weak arrivals is made on the travel-time chart, and gives an indication of the confidence with which a given arrival can be recognized on the seismogram. The desirability of giving extra weight to strong arrivals was considered, but was rejected on the grounds that their production might be favoured by certain types of geological irregularity (§ 5 below).

(b) *Correction for unconsolidated sediments*

At the distances with which the present investigation is concerned, the seismic waves travel for most of their path in or below the 'basement complex' which underlies the upper sediments. In order that their passage may be detected, the waves must be refracted upwards through the sediments, and the time taken to reach the recording instrument will contain a term dependent on the thickness and nature of the upper layers. In north Germany and the Low Countries this term is of particular importance, as the harder rocks are covered by a great thickness of glacial and alluvial drift, with a propagation velocity of about 1·7 km./sec.

An exact determination of the effect of the drift would require a knowledge of the variation of velocity with depth below every one of the recording stations, but an approximate correction can be expressed in terms of the time taken for the shock of a small explosion to reach a seismograph at some standard distance along the surface.

Consider a seismograph at a distance D_0 from the small explosion, and assume that the first wave to arrive at the seismograph reaches a maximum depth H, where the velocity is v_2. Let the velocity at all points less than H be v_1, where $v_1 < v_2$. Then the travel time is given by

$$T = \frac{D_0}{v_2} + \frac{2H[1-(v_1/v_2)^2]^{\frac{1}{2}}}{v_1}. \tag{1}$$

Now consider a wave arriving at the same seismograph from a source at a distance $D > D_0$. If this wave penetrates to a depth H_1 where the velocity is v_3, its travel time will be given by

$$t = \frac{D}{v_3} + \frac{H[1-(v_1/v_3)^2]^{\frac{1}{2}}}{v_1} + \int_H^{H_1} \left[\frac{1}{v^2} - \frac{1}{v_3^2}\right]^{\frac{1}{2}} dz + I,$$

where v is the velocity at a depth z between H and H_1 and I is a term depending on the rocks near the source.

If the material at depths less than H were replaced by rock with velocity v_2, the time would become

$$t' = \frac{D}{v_3} + \frac{H[1-(v_2/v_3)^2]^{\frac{1}{2}}}{v_2} + \int_H^{H_1} \left[\frac{1}{v^2} - \frac{1}{v_3^2}\right]^{\frac{1}{2}} dz + I.$$

The correction which is required to convert the observed time t to the value which would be obtained if the drift were to be so replaced is

$$t'-t=+H\left\{\frac{[1-(v_2/v_3)^2]^{\frac{1}{2}}}{v_2}-\frac{[1-(v_1/v_3)^2]^{\frac{1}{2}}}{v_1}\right\}$$

or, using (1), $$t'-t=\frac{1}{2}\left\{T-\frac{D_0}{v_2}\right\}\left\{\left[1-\left(\frac{v_2}{v_3}\right)^2\right]^{\frac{1}{2}}\frac{v_1}{v_2}-\left[1-\left(\frac{v_1}{v_3}\right)^2\right]^{\frac{1}{2}}\right\}\left\{1-\left(\frac{v_1}{v_2}\right)^2\right\}^{-\frac{1}{2}}. \tag{2}$$

In the course of the geophysical survey of the north German plain, values of T for $D_0=4$ km. had been determined at a large number of points. These values were incorporated in the 'travel-time plan' of the area (*Geologisches Landes Amt.* 1947). Times taken from this plan have been used to apply corrections to the waves which we call P_s, P_g and P_n, which have velocities of about 4·4, 5·5 and 8·2 km./sec. respectively. v_2 has been taken as 4 km./sec, and v_1 is about 1·7 km./sec. For these velocities, (2) gives

$$t'-t=-(T-1)\,A, \tag{3}$$

where A is a numerical constant whose value is 0·41, 0·36 and 0·33 for P_s, P_g and P_n respectively. Variations of 20 % in v_1 and v_2 change A by about 5 %, but the variations in v_2 have a larger effect on the term $[T-D_0/v_2]$. The latter effect is, however, the same for all the stations, and the choice of v_2 simply fixes the arbitrary depth to which we attempt to extend the correction. In general, the maximum velocity for the first arrival at 4 km. will differ from the assumed value of v_2, and an accurate correction would require T to be defined as the travel time along the ray which reaches the assumed velocity at its lowest point. The time for the first arrival will usually be less than the true value of T, and the method therefore applies a conservative correction.

In north Germany, the travel times for 4 km. vary from 1·6 to 2·1 sec. and the corrections from $-0·2$ to $-0·4$ sec. An additional correction with $T=1·85$ sec. was applied to the Soltau results to allow for the drift below the shot point. The corrections may therefore be as much as four times the standard deviation of P_n.

(c) *Interpretation of travel times along short lines*

In accordance with the usual procedure for determining structures, the times required for a wave to reach the successive seismographs in a given line have been expressed in terms of its apparent velocity and time of starting. This involves the assumption that the travel-time curve consists of a series of straight lines. The most general structure whose properties can be determined from such a curve consists of uniform layers of rock separated by plane boundaries. If the layers are inclined, the apparent velocity of the waves will depend on the direction of propagation, but the intercepts of the various branches of the curve will be determined only by the structure immediately below the shot point. In the present study, the variation of velocity with azimuth is less regular than would be expected if a single system of plane layers were to extend under all the stations, and significant differences are found to occur between the apparent times of starting along the various profiles. For a given line of stations significant departures from linearity are comparatively rare. We are therefore led to the idea of a structure for which the deviations from plane stratification become important at distances of the order of tens of kilometres.

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The uncertainties which arise in this way are most serious when the length of the profile and the distance separating the nearest observing station from the shot point are both comparable with the scale of the structural variations. The velocities of the waves over the observed and unobserved sections of the path may then differ significantly from each other, and any such difference will contribute to the apparent delay of starting. The contribution will be positive if the velocity near the shot point is less than that below the stations. This condition is likely to arise if the stations record an exceptionally high velocity, so that we may expect to find a positive correlation between the velocities and starting times in the various directions. The difficulty has not been considered in natural earthquake studies because the separation between the stations is usually sufficient to ensure that the local variations appear as independent residuals. The differences between the travel times for the groups of stations in the present study are substantially smaller than the residuals in ordinary earthquake work, but they are thrown into significance by the consistency within each group.

(*d*) *The significance of doubtful lines*

The open time scale of the present series of seismograms often enables arrivals to be resolved on a single record when the interval between them is only 1 or 2 sec., and we are therefore presented with the opportunity of tracing phases back into congested regions of the travel-time chart. This process involves the identification of phases whose separation at a given station is comparable with the residuals of a given phase from its travel-time curve. This introduces a danger of gross errors, for it is possible to use the method of least squares to fit a curve to a completely arbitrary selection of points, provided only that the number of parameters required to specify the curve is less than the number of points chosen. The residuals can then be used to estimate the standard deviation of the observations, and the uncertainties of the parameters can be calculated. If the points are chosen from a dense population on the grounds that they are close to a preconceived line the estimated uncertainties may be quite small, even if the original population bears no relation to the law expressed by the line. If the points are distributed at random, the number whose residuals are less than any specified value will be proportional to the area of the chart which falls within the specified range about the line.

Suppose now that a second curve is drawn, whose equation expresses a law which approximately connects the co-ordinates of some of the points. These points will cluster along the second curve, and the number of points within a given band width will tend to exceed the number which lie within an equal area in other parts of the diagram. The extent to which the observations uphold the existence of the law will be measured by the significance of the excess concentration in the neighbourhood of the curve. The sensitivity of the test will depend on the choice of band width, for excessively narrow limits will reduce the sample available for comparison, whilst wider ones reduce the apparent density by averaging it over a greater area of the chart. If the points governed by the law are distributed normally about the curve most of them will fall within a range of $\pm\sigma$, and trial calculations have suggested that this band width does give the greatest significance in the presence of a uniform background.

The procedure for testing the significance of a line is therefore as follows:

(i) Count the total number of observations within the area under investigation, and hence estimate the mean density of arrivals on the travel-time chart.

(ii) To test the extent to which a selected group of these observations support a given law, estimate the standard deviation of the selected observations about the line which represents the law. Count the total number of observations which lie within $\pm s$ of the line (where s is the best available estimate of σ). This number must necessarily exceed the number of degrees of freedom used in fitting the line, so we define the *successes* of the hypothesis as the number of observations within $\pm s$, minus the number of degrees of freedom expended.

(iii) The number of successes is compared with that which would be expected if all the observations were distributed at random over the chart.

In practice, the accuracy with which the significance can be assessed is limited by the uncertainty in the background density. Many of the observations are clearly connected with well-defined lines, but it is assumed that they will still behave like a random distribution with respect to other lines. In view of the crudity of this assumption the method gives only a rough indication of the absolute significance of an individual line, but may be expected to fare better when it is used to decide between alternative hypotheses for which the travel-time curves are only slightly different.

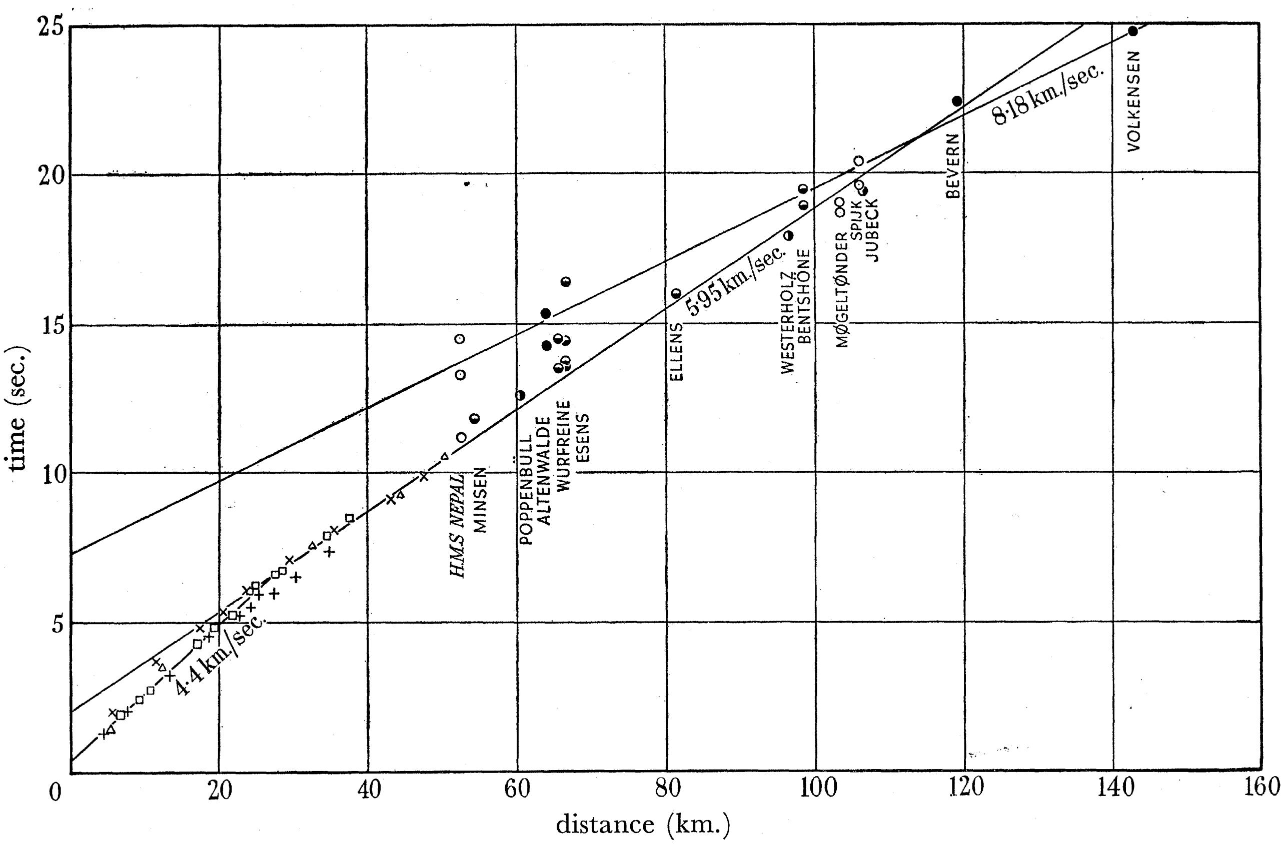

FIGURE 2. Travel times at near stations. Symbols: + Soltau W. profile, × N.W. profile, □ S.W. profile, △ E. profile. Heligoland stations: ◑ Schleswig line, ◒ Oldenburg line, ● Göttingen line, ○ Other stations. To reduce the congestion at short distances, some of the Soltau points have been omitted.

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4. Times of travel

The times at which the seismic waves arrived at the stations are given in table 1 and are plotted in figures 2 and 3. The first waves to arrive are the easiest to distinguish, and are therefore subjected to a more detailed examination than the others. For a given line of stations, it is found that the times of the first arrivals can be fitted on to a small number of straight lines, which implies that the waves can be divided into a number of phases, each having a characteristic velocity of propagation. These velocities are given in table 2.

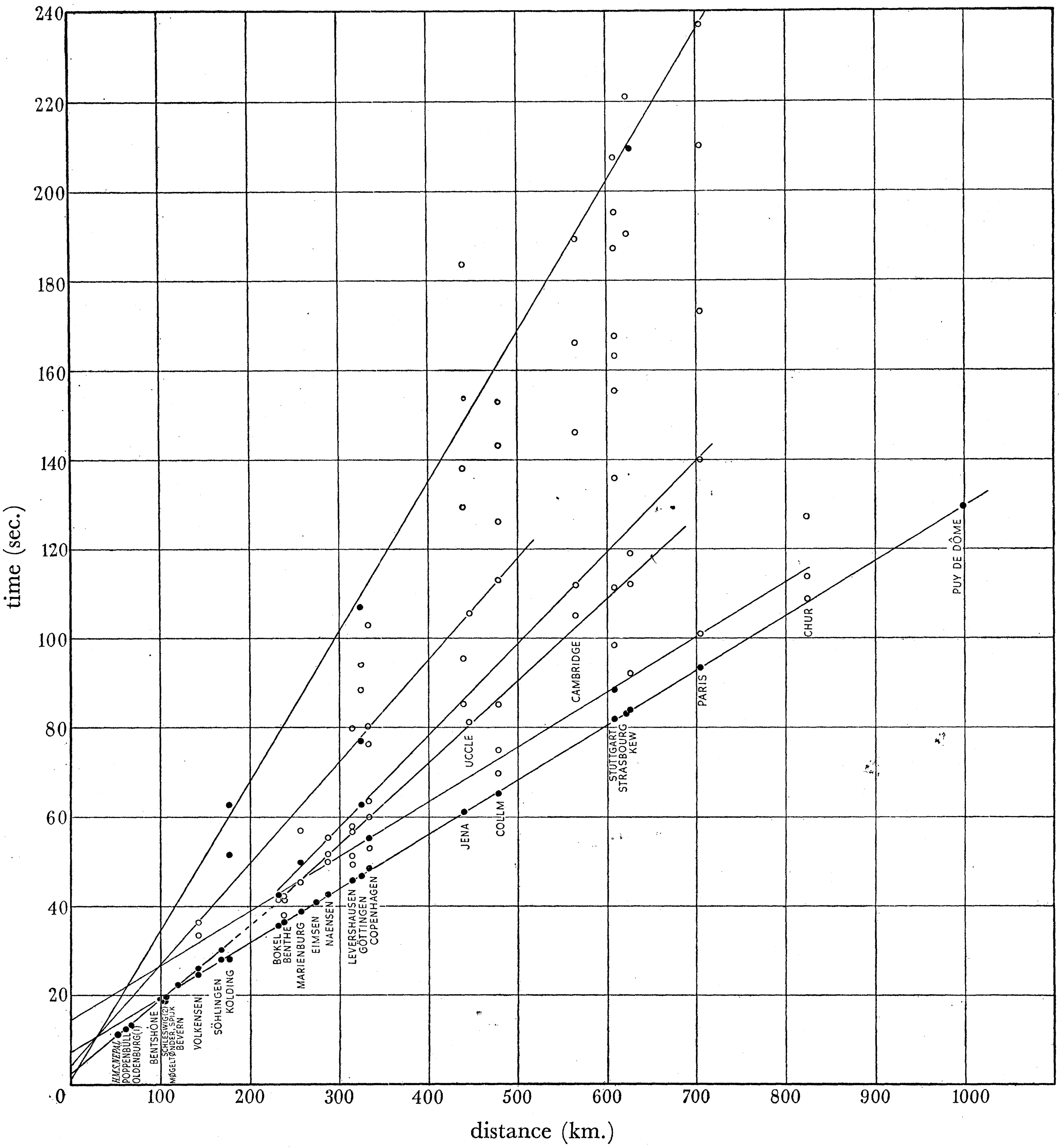

Figure 3. Travel times at longer distances. Symbols: ● Strong onsets, ○ weaker onsets.

Table 1. Times of arrival of seismic waves

(*a*) Soltau stations. First arrivals only

station	no.	profile	range (km.)	onset time	phase	correction	residual (sec.)
Schneverdingen, E.	20	W.	4·62	2·00	P_s	0·70	−0·05
Heber	1+59	S.W.	5·10	2·21	P_s	0·70	+0·05
Behringen	54	E.	5·21	2·13	P_s	0·7*	−0·05
Barl	13	N.W.	5·85	2·62	P_s	0·62	+0·37
Steinkenhöfen	64	E.	6·40	2·46	P_s	0·7*	0
Schneverdingen, W.	29	W.	6·55	2·41	P_s	0·69	−0·08
Steinkenhöfen	66	E.	6·63	2·64	P_s	0·7*	+0·06
Surbostel	60+36	S.W.	6·85	2·61	P_s	0·71	+0·04
Zahrensen	21	W.	7·90	2·70	P_s	0·68	−0·08
Wolterdingen, N.	61 and 37	S.W.	9·26	3·12	P_s	0·72	0
Iserhatsche	65	E.	10·38	3·61	P_s	0·7*	+0·25
Wolterdingen, S.	2+38	S.W.	10·80	3·50	P_s	0·72	+0·03
Lünzen	22	W.	10·90	3·26	P_s	0·68	−0·20
Wesseloh	8	N.W.	11·65	4·41	P_s	0·70	+0·76
Steinbeck	56	E.	12·29	4·17	P_s	0·7*	+0·38
Wiedingen	39	S.W.	12·80	4·03	P_s	0·70	+0·17
Bult	23	W.	13·30	3·89	P_s	0·67	−0·10
Leitzingen	40	S.W.	17·10	5·05	P_s	0·72	+0·17
Schillingsbostel	9	N.W.	17·53	5·60	P_s	0·72	+0·60
Hemslingen, N.E.	30	W.	17·65	4·81	P_s	0·57	−0·06
Hemslingen, W.	31	W.	18·70	5·28	P_s	0·70	+0·03
Grosseholz	3	S.W.	19·60	5·61	P_s	0·74	+0·17
Todtglüsingen	14	N.W.	20·30	6·13	P_s	0·76	+0·45
Avenriep	41	S.W.	21·65	5·98	P_s	0·74	+0·02
Brockel, E.	32	W.	22·80	6·00	P_s	0·76	−0·24
Dohren	10	N.W.	23·25	6·91	P_s	0·76	+0·54
Brockel, W.	24	W.	24·10	6·14	P_g	0·62	−0·47
Riepe	4	S.W.	24·10	6·66	P_s or P_g	—	—
Westerhorn	68	E.	24·41	6·79	P_s or P_g	—	—
Dorfmark	z	S.W.	25·10	6·90	P_g	0·65	+0·10
Wensebrook	33	W.	25·60	6·59	P_g	0·61	−0·25
Hemsbünde	25	W.	27·30	6·63	P_g	0·60	−0·48
Vierde	42	S.W.	27·35	7·28	P_g	0·61	+0·15
Adolphsheide	5	S.W.	28·50	7·38	P_g	0·62	+0·04
Holtorf	15	N.W.	29·60	7·82	P_g	0·69	+0·23
Rotenburg, E.	26	W.	30·20	7·13	P_g	0·57	−0·44
Regesbostel	12	N.W.	32·43	8·19	P_g	0·66	+0·15
Ellerndorf	70	E.	32·47	8·17	P_g	0·55	+0·23
Düshorn, N.	43	S.W.	34·30	8·40	P_g	0·54	+0·17
Waffensen	35	W.	34·90	7·98	P_g	0·59	−0·40
Goldbeck	16	N.W.	36·20	8·78	P_g	0·67	+0·10
bei Kreglingen	44	S.W.	37·50	8·97	P_g	0·50	+0·24
Grundoldendorf	17	N.W.	43·10	9·78	P_g	0·66	−0·05
Hansen	72	E.	44·65	9·89	P_g	0·59	−0·13
Horneberg	18	N.W.	47·50	10·50	P_g	0·67	−0·08
Hambrock	73	E.	50·30	11·18	P_g	0·60	+0·20

(*b*) Heligoland stations

station	lat.	long.	range (km.)	onset time	phase	sedimentary correction	residual (sec.)	comments
H.M.S. *Nepal*	54° 16·6′	7° 5·5′	52·6	11·2	P_g	—	−0·07	
				13·3				
				14·5				
				26·8	—	—	—	very strong phase
				29·8				
				32·0				
				36·5				
Minsen	53° 41·8′	7° 59·2′	54·2	12·15	P_g	−0·34	+0·28	
				24·8				
				33·0				
				43·6				

For explanation of symbols see end of table.

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Table 1 (*cont.*)

(*b*) Heligoland stations (*cont*).

station	lat.	long.	range (km.)	onset time	phase	sedimentary correction	residual (sec.)	comments
Poppenbüll	53° 21·7′	8° 45·7′	60·3	13·01	P_g	0·37	+0·08	
Altenwalde	53° 50·2′	8° 40·2′	63·9	14·55	P_g†	0·34	+0·94	feeble onset obscured by electrical disturbance
				15·67	P_n†	0·31	+0·18	
Wurfreihe	53° 35·6′	7° 59·2′	65·6	13·8	P_g	0·35	+0·04	
				14·8	P_g†	0·35	+1·04	
				33·4				
Esens	53° 37·2′	7° 31·9′	66·6	13·83	P_g	0·29	−0·07	
				14·09	P_g†	0·29	+0·09	
				14·70	P_n†	0·27	−1·07	
				16·61				
Ellens	53° 27·2′	8° 0·5′	81·2	16·56	P_g	0·32	+0·17	
				49·50				
Westerholz	54° 29·5′	9° 16·3′	96·5	18·33	P_g	0·37	−0·33	
Bentshöhe	53° 17·9′	8° 0·9′	98·3	19·3	P_g	0·32	−0·03	
				19·8	P_n†	0·29	+0·10	
				66·0				
Møgeltønder	54° 56·4′	8° 48·9′	103·7	19·0	P_g	0·29*	−1·13	
				19·34	P_n†	0·26*	−0·92	
Spijk	53° 25·7′	6° 32·3′	106·0	20·0	P_g	0·32*	−0·55	
				20·7	P_n†	0·29*	+0·05	
				35·8	R†		−0·20	
Jübeck	54° 32·4′	9° 24·2′	106·3	19·81	P_g	0·38	−0·48	
Bevern	53° 25·7′	9° 11·0′	119·1	22·75	P_g or P_n	0·29	−0·07	included in P_g reduction
Volkensen	53° 16·4′	9° 25·2′	142·4	25·0	P_n	0·24	0	
				26·45	P_g	0·27		
				33·66	P_x†	0·24	+0·94	
				36·66	S_n†	0·30	−0·2	correction computed by assuming that S waves are refracted upwards as P
Astrid	54° 57′	5° 55′	153·6	27·4	P_n†	—	+1·2	small sharp onsets similar in appearance to spurious disturbance (possibly ship noise)
				41·5	S_n†	—	+2·3	
				51·4	R†	—	−2·4	
Söhlingen	53° 4·6′	9° 36·9′	167·7	27·98	P_n	0·07	+0·07	
				30·35	P_g	0·07		
Kolding	55° 28·8′	9° 28·7′	177·0	28·39	P_n	0·22	−0·83	
				52·02				
				62·45	R	—	+1·2	
Bokel	52° 48·4′	10° 31·5′	232·3	35·75	P_n	0·12	−0·15	
				41·5	P_g	0·15		
				41·65	P_g			
Benthe	52° 18·5′	9° 37·5′	238·4	36·51	P_n	—	0·03	
				38·02	P*	—	0	
				39·80				
				41·30				
				42·20	P_x†	—	1·65	
Marienburg	52° 10·4′	9° 46·1′	256·9	38·61	P_n	—	−0·13	
				45·4	P_x	—	+0·71	
				50·0				
				56·86				
Eimsen	52° 0·4′	9° 49·2′	274·3	40·8	P_n	—	−0·06	
Naensen	51° 53·9′	9° 53·5′	287·3	42·5	P_n	—	0·04	
				49·9	P_x	—	0·07	
				51·6				
				55·3				
Levershausen	51° 39·8′	10° 0·3′	314·1	45·9	P_n	—	—	no radio; zero measured from P_n onset
				49·5	P*		−0·33	
				51·4				
				56·6				
				58·0				
				79·7				
Göttingen	51° 32·8′	9° 58·0′	324·8	46·7	P_n	—	−0·30	
				62·9	P_g			
				77·5	S_n	—	−0·60	
				88·4				
				92·1				
				108·8	R	—	−1·45	

TABLE 1 (*cont.*)

(*b*) Heligoland stations (*cont.*)

station	lat.	long.	range (km.)	onset time	phase	sedimentary correction	residual (sec.)	comments
Copenhagen	55° 41·2′	12° 27·0′	333·2	48·7	P_n	0·22	+0·44	
				53·2	P^*	0·23	+0·12	
				55·4	P_x	0·22	−0·26	
				60·1 }	—	—	—	resolution of onsets doubtful
				63·81 }	P_g			
				80·75	S_n	0·28	+0·6	
Jena	—	—	439·5	61·0	P_n	—	−0·06	
				85·5	P_g			
				95·5				
				138				
				154				
				183·5				
Uccle	50° 47·9′	4° 21·5′	446·3	81·0	P_g			
				105·5	S_n	—	−0·2	
Collm	—	—	470·8	65·0	P_n	—	+0·11	
				69·3				
				75·0	P^*	—	+0·54	
				84·5				
				111·5	S_n	—	+0·1	
				126·0				
				143·0				
				153·0				
Cambridge	52° 12·9′	0° 5·8′	564·9	104·9	P_g			
				111·8				
				146·0				
				166·0				
				189·4	R	—	−0·8	
Stuttgart	48° 46·25′	9° 11·6′	608·2	81·8	P_n	—	+0·12	
				88·2	P_x	—	−0·52	
				98·1	P^*	—	+2·10	
				111·3	P_g			
				135·7				
				155·3				
				161·5				
				167·7				
				187·5				
				195·5				
				207·5	R	—	+1·9	
Strasbourg	48° 35·1′	7° 45·9′	622·4	83·0	P_n	—	−0·31	
				190·5				
				220·5				
Kew	51° 28·1′	−0° 18·8′	626·1	84·0	P_n	—	+0·13	
				92·0	P_x	—	+0·6	
				112·0	P_g			
				119·0				
				209·5	R	—	−1·1	
Paris	48° 48·6′	2° 29·6′	704·8	93·4	P_n	—	−0·14	
				100·9	P_x	—	0	
				140·0	P_g			
				173·2				
				210·0				
				237·0	R	—	+0·3	
Chur	46° 51·0′	9° 32·2′	823·3	108·7	P_n†	—	+0·64	
				113·9	P_x	—	−1·4	
				127·3	P^*	—	−2·4	
Puy de Dôme	45° 46·5′	2° 58·0′	998·0	129·4	P_n	—	+0·08	
				242·3				
				324·5				

Values of the sedimentary correction marked with an asterisk are estimates at the stations which were not covered by t available sheets of the geotechnic map.

† distinguishes onsets which have not been included in the computation of travel-time curves.

P_g residuals at the near stations are values of (I_2-I_0), as given in § 4 (*e*).

Residuals for P_g at distant stations have not been included, because of the difficulty of interpreting this phase (§ 4 (*e*

The numbers of the Soltau stations correspond with those given in the Göttingen report, from which the results were tak

TABLE 2

shot point	direction of line	range (km.)	velocity (km./sec.)	intercept (sec.)	phase
Soltau	E.	5·2– 12·3	3·54 ± 0·1	−0·01 ± 0·08	P_s?
	S.W.	5·0– 21·7	4·34 ± 0·06	0·34 ± 0·04	P_s
	W.	4·6– 22·8	4·43 ± 0·075	0·22 ± 0·05	P_s
	N.W.	5·9– 23·3	3·7 – 4·9	—	P_s?
	E.	32·5– 50·3	6·0 ± 0·8	2·1 ± 1·0	P_g
	S.W.	24·1– 37·5	5·60 ± 0·06	1·79 ± 0·05	P_g
	W.	24·1– 34·9	6·3 ± 0·5	1·8 ± 0·4	P_g
	N.W.	29·6– 47·5	6·65 ± 0·07	2·67 ± 0·06	P_g
Heligoland	E.	60·3–106·3	6·78 ± 0·03	3·9 ± 0·07	P_g
	S.	54·2– 98·3	6·0 ± 0·2	2·65 ± 0·37	P_g
	all distant stations	120–998	8·18 ± 0·014	7·37 ± 0·013	P_n

(*a*) *Nomenclature of phases*

The velocities can be divided into groups according to the distance from the shot point at which they are observed. The differences between stations in the same range of distance are not usually sufficient to disturb the classification, but in doubtful cases phases would be separated if the waves were believed to travel in layers at different depths and grouped together if they were travelling in different blocks of material at the same depth. In the range from 4 to 24 km. from Soltau, the waves having velocities of 4·43 and 4·34 km./sec. are called P_s. The wave travelling at 4·9 km./sec. along the north-west profile is included in P_s, but the classification of the ones which travel at about 3·7 km./sec. will require further discussion. Between 25 and 120 km. the velocities cover the range 5·6 to 6·78 km./sec. and the waves are called P_g. The possibility that some of these waves might correspond to the P^* of near earthquakes is discussed, but this distinction would not be made on the basis of the Heligoland results alone.

Beyond 120 km. the first waves to arrive are called P_n. A wave which corresponds roughly with the P_g of near earthquake work is recorded after P_n, and the resolution of further phases between these two is discussed in detail. Considerable disturbed motion occurred during the expected times of arrival of S waves, but the individual phases could not be identified with certainty.

(*b*) 4 *to* 24 km.

This range of distance is covered by the four Soltau profiles. The travel times along the west and south-west lines are in fairly close agreement, but along the east and north-west profiles the waves travel more slowly, and are 0·4 sec. behind the others at a distance of 12 km. from the shot point. There are no observations between 12 and 24 km. along the east profile, but at this distance along the north-west profile the waves start to overhaul those travelling to the west and south-west.

The waves observed at the beginning of the east and north-west profiles have been considered in the light of the following hypotheses:

(*a*) That they are transmitted through the same material as those to the west and south-west, and that the difference of apparent velocity arises from an inclination of the strata.

(*b*) That they are transmitted through a separate layer lying above that which transmits P_s.

(*c*) That they are transmitted through a block of rock lying beside the others, and forming part of the same complex.

The decision between (*a*) and (*b*) would be important in a study of the geology of the upper layers, but the present work is concerned mainly with waves refracted up from greater depths. From this point of view it is sufficient to point out that both hypotheses require the existence of a layer above the one which transmits P_s, and thick enough to cause the observed delays. In fact, the data will not decide between (*a*), (*b*) and (*c*), and in assigning an average velocity to P_s we have no means of choosing between a single layer whose velocity ranges from 3·5 to 4·9 km./sec. and a structure with a velocity of less than 3·5 km./sec. near the surface, and 4·3 to 4·9 km./sec. at depth. For convenience in the ensuing calculations an arbitrary velocity of 4·4 km./sec. will be used.

(*c*) 24 *to* 106 km.

At distances beyond 24 km. the velocity of the first arrivals is always greater than 5·6 km./sec., and the apparent delays of starting greater than 1·79 sec. The transition from P_s is clearly observed on all four of the Soltau profiles, and there are a few cases where P_s is observed as a second arrival.

Of the velocities obtained in the Soltau experiments, the one along the north-west profile stands out as anomalous both in velocity and intercept. The anomaly can be regarded as a delay in the arrivals between 29 and 36 km. from the shot point, and as such would fit in with the hypotheses (*a*) and (*b*) of P_s. In any case, the high velocity throws doubt on the interpretation of the intercept for this profile. The estimates for the thickness of the P_s layer obtained from the east, south-west and west profiles are 6·9, 6·4 and 5·6 km. respectively, and the apparent uncertainties are such that the weighted mean would only slightly exceed 5·6 km. Since, however, the standard deviations of the numerical constants do not include all the sources of error, the small variance along the west profile should not be allowed to increase the weight of the result to the full apparent value. The low velocity observed for P_s at the beginning of the east profile suggests that the estimate of thickness in this direction may be too great. Taking these factors into account, we shall assume a depth of 6·0 km. below Soltau, and allow an uncertainty of about 0·5 km.

The velocities observed along the two Heligoland profiles are also inconsistent with each other, but the high value to the east had been expected by the Germans. The stations in this region run across an area where the magnetic anomaly changes from -150 to $+250\gamma$ and it is thought that the material which causes the anomaly is likely to have a high propagation velocity. In deciding the most appropriate values for the average velocity and depth of propagation of P_g, it will be necessary to include the observations from the isolated stations of Spijk, Møgeltønder, Bevern and H.M.S. *Nepal*. The residuals at neighbouring stations will not be independent unless their separation is large compared with the extent of the regional anomalies, so that the inclusion of all the separate travel times in the reduction may give excessive weight to the most densely populated regions. In order to attach more nearly equal weight to equal areas of the ground, the onset times at the densest station groups were averaged, and the group means treated as single observations in the ensuing calculation. The first four stations on the Oldenburg line were grouped together, as were the last two stations in Schleswig-Holstein. The resulting equation for P_g from Heligoland is then

$$t = 3{\cdot}15+\Delta/6{\cdot}38\ \text{sec.} \tag{4}$$

On including the group means of P_g along the four Soltau profiles, the lumped equation for the two shot points becomes

$$t = 2{\cdot}06 + \Delta/5{\cdot}95 \text{ sec.} \tag{5}$$

The group means and their deviations from the latter line are given in table 3.

TABLE 3

station	Δ	t	residual
Soltau: W. profile	28·4	6·30	−0·54
S.W. profile	29·5	7·01	0
N.W. profile	37·8	8·34	−0·06
E. profile	42·4	9·17	−0·02
H.M.S. *Nepal*	52·6	11·20	+0·31
Poppenbüll	60·3	12·64	+0·45
Oldenburg (1)	66·9	13·77	+0·48
Bentshöhe	98·4	18·98	+0·40
Schleswig (2)	101·4	18·69	−0·40
Møgeltønder	103·7	18·71	−0·76
Spijk	106·0	19·68	−0·18
Bevern	119·1	22·46	+0·40

Poppenbüll is at the beginning of the Schleswig line, and Bentshöhe at the end of the Oldenburg one. They could have been called Schleswig (1) and Oldenburg (2) respectively. The result from Altenwalde was not included, as the first arrival was feeble and obscured by heavy electrical disturbance.

If the differences in velocity corresponded to lithological differences in the materials of the P_g layer, the argument could not be carried much further. If, however, the residuals are supposed to arise mainly from delays in penetrating the P_s layer, they can be used to give a direct indication of the conditions below each station. The time required for the wave to reach a given station will be given by

$$t_{\text{obs.}} = \frac{D}{v_3} + \int_0^{H_1} \left[\frac{1}{v^2} - \frac{1}{v_3^2}\right]^{\frac{1}{2}} dz + \int_0^{H_2} \left[\frac{1}{v^2} - \frac{1}{v_3^2}\right]^{\frac{1}{2}} dz$$

$$= \frac{D}{v_3} + I_1 + I_2, \tag{6}$$

where H_1 and H_2 are the thicknesses of the sediments under the shot point and seismograph. The time calculated from the average line (combining data from all shot points) can be written

$$t_{\text{calc.}} = \frac{D}{v_3} + 2I_0. \tag{7}$$

The residual for any individual station is then

$$t_{\text{obs.}} - t_{\text{calc.}} = (I_1 - I_0) + (I_2 - I_0). \tag{8}$$

Equation (7) defines I_0, and a value of I_1 can be estimated for each shot point. Equation (8) then defines I_2, which is the intercept of the travel-time curve which would be obtained from a series of explosions at various distances from the station. These intercepts may be converted into the thicknesses of a supposedly uniform P_s layer. The velocities given above yield a conversion factor of 6·51 km./sec.

If table 3 is inspected in the light of the foregoing argument, it will be noted that the P_g interface below Møgeltønder, Jubeck and Spijk, lies above the mean level surface, but that it falls below that level at all points within 70 km. of Heligoland. It therefore appears that Heligoland lies near the centre of a trough in the P_g layer. It is unlikely that the dips between the coast and the island are as steep as they are further towards the sides of the depression, so that the extrapolation involved in the constant term of equation (4) will probably overestimate the thickness of sediments. On the other hand, we should expect the depth under Heligoland to be greater than those under H.M.S. *Nepal* and Poppenbüll. These limits locate the intercept of the Heligoland P_g curve between 2·5 and 3·15 sec., and a value of 2·8 sec. will be assigned. The corresponding value for Soltau is 1·84 sec.

It is now possible to calculate $(I_2 - I_0)$ for each station from equation (5), and it is these which have been entered in the column of residuals in table 1. The depth of the upper surface of the P_g layer is entered in brackets by each station on figures 1 *a* and *b*.

The assumptions involved in this calculation are admittedly somewhat arbitrary, but there are a few points at which the results can be checked independently. At Söhlingen, the depth estimated from the extrapolated P_g line is 5·1 km., which fits fairly closely with the values along the Soltau west profile. The negative depth for Møgeltønder is, of course, an exaggeration, but short-range seismic work in this area has revealed rocks with a velocity of 5·1 km./sec. within 1·7 km. of the surface (Nørlund & Brockamp 1934). This velocity is higher than any of the P_s values near Soltau, and would help to account for the short time of travel. The Danish station at Kolding gives a residual of $-0{\cdot}83$ sec. on P_n, and thereby provides an indication of crustal thinning not far from Møgeltønder, without depending so strongly on the velocity of P_g. Finally, the gravity surveys of northern Germany (Kossmat 1931) and Denmark (Nørlund 1930) show Bouguer anomalies of +15 to 20 mgals near Westerholz, Jubëck and Møgeltønder, +0 to 5 mgals for Oldenburg and Poppenbüll, −3 for Bevern and −12 for Heligoland. These give general support to the idea of a depression below the island.

(*d*) *The possibility of an extra layer*

It may be argued that the distribution of residuals in table 3 suggests an increase of velocity with distance from the shot point, and hence that some of the Heligoland arrivals represent a wave travelling faster than P_g in a deeper layer. It is most unfortunate that the Soltau waves were never observed at more than 50 km. or the Heligoland ones at less than that distance. In the absence of such a direct check any argument must be inconclusive, and we should note that some of the Soltau velocities exceed the one found in Oldenburg, and that the chief differences are between the constant terms. The introduction of the extra layer would do nothing to explain the large differences between the Soltau west and south-west groups, or between Møgeltønder and Bentshöhe, and the observations of second arrivals are too erratic to be of any assistance.

(*e*) P_g *and* P_s *at distant stations*

On referring to figure 3, it will be observed that the stations from Bokel to Paris record a prominent group of arrivals, for which the velocity is a little lower than that of P_g at the near stations. Three particularly strong arrivals are indicated by the black points from Marienburg, Göttingen and Jena, but a number of weaker onsets are clustered about a line between 5 and 10 sec. earlier. The appearance of the chart suggests that more than one phase may be present, but the form of the distribution does not show at once how the lines should be drawn, or how the points should be divided between them.

In order to reduce the results to a more convenient form, a straight line was fitted to the arrivals which appeared on the first inspection to fall into the lower group. The equation of this line is

$$t = -0{\cdot}6 + \Delta/5{\cdot}5 \text{ sec.},$$

and the residuals of points near the line are plotted on figure 4.

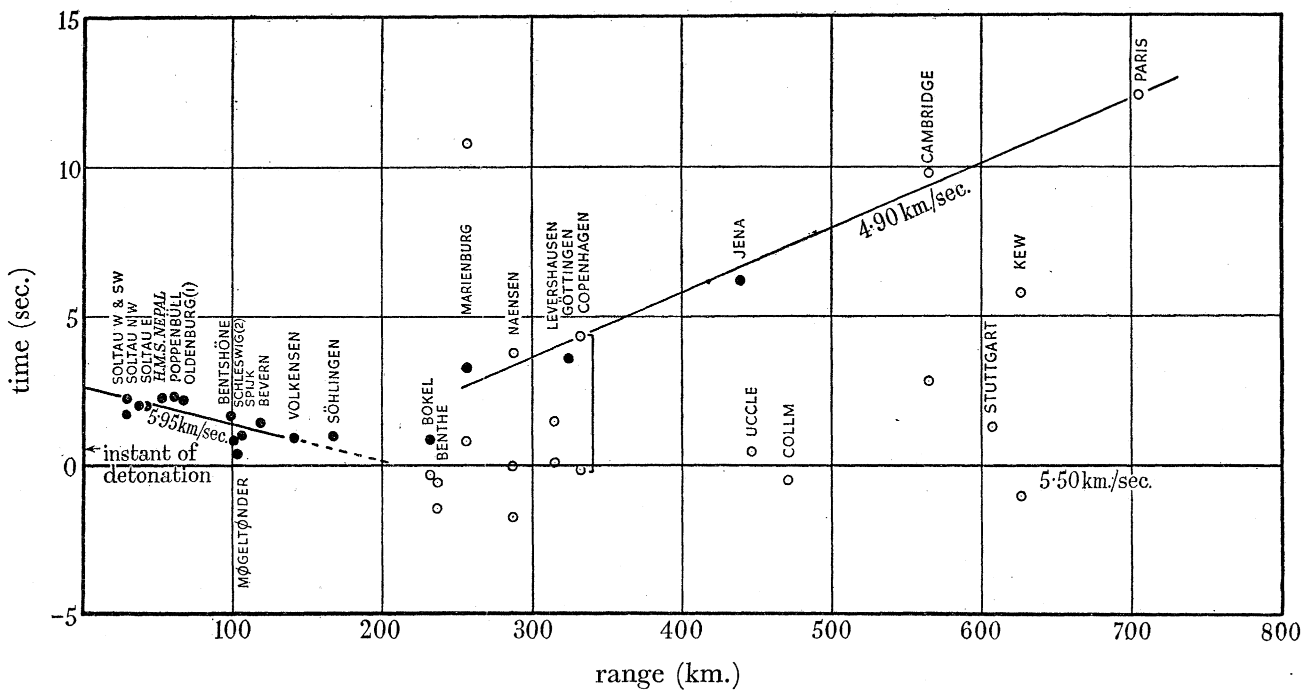

Figure 4. Residuals from P_g lower line. Symbols as in figure 3.

It is fairly evident that the observations are not grouped normally about any one line, but the observed distribution could arise from any of the following causes:

(*a*) That the waves are transmitted through two separate layers.

(*b*) That all the waves travel for most of their path in a single layer, but that the later ones are refracted up to and reflected back from the surface at one or more points between the shot point and seismograph.

(*c*) That the late arrivals are transmitted along slow paths through a layer of variable constitution.

(*d*) That there is a tendency for arrivals to be read late rather than early when they grow out of a disturbed background.

Hypotheses (*a*) and (*b*) are suitable for comparison by the method of § 3 (*d*). There are twenty-two arrivals spread over a range of about 14 sec., corresponding to a density of 1·5 observations per sec. The lower line is common to both hypotheses and the standard deviation of the selected points about it is 0·86 sec. A total of nine observations fall within $\pm s$ of the line, and two degrees of freedom have been expended. The line therefore scores seven successes against an expectation of 2·6.

On hypothesis (*a*), it is natural to fit all the black points on to the second line, and this suggests that the arrivals at Cambridge and Paris should also be included. The standard deviation is 0·5 sec. and four observations fall within $\pm s$. The line therefore scores only two successes against an expectation of 1·5. On hypothesis (*b*) the slope must be the same for both lines and only three degrees of freedom are expended. This will be regarded as an expenditure of 1·5 degrees of freedom per line. When the arrivals at Cambridge and Paris are omitted and the one at Kew is introduced, the common velocity becomes 5·43 km./sec. and the upper line scores 3·5 successes against an expectation of 3.

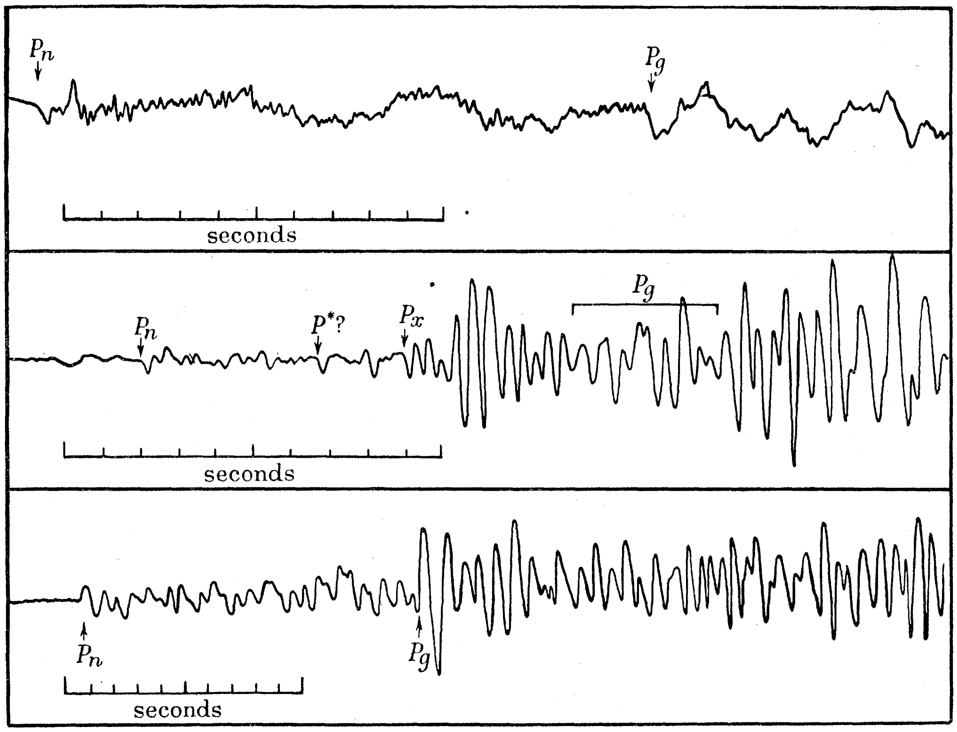

FIGURE 5. Tracings of seismograms from (*a*) Göttingen, 324·8 km., (*b*) Copenhagen, 333·2 km., and (*c*) Jena, 439·5 km. The tracing from Jena was taken from a rather blurred photographic record, and the minor irregularities may be unreliable.

The form of distribution to be expected from hypothesis (*c*) will depend on the arrangement of the lithological boundaries. There seems to be no tendency for the late arrivals to be recorded in any particular azimuth, and a number of single stations record double shocks. This suggests an array of fairly small blocks of material, and the scarcity of onsets towards the centre of the distribution might arise from the difficulty of reading a second arrival if it follows closely behind an earlier one.

Hypothesis (*d*) has been considered because skew distributions have sometimes been recorded in earthquake work, but they have always been associated with doubtful readings of weak onsets. In the present case, the great strength of the arrivals at Göttingen and Jena excludes the possibility that errors of reading could exceed a few tenths of a second (see figure 5).

The decision between the surviving hypotheses cannot be made with certainty, but some difficulties are associated with the acceptance of (*a*). The upper line in this case is presumably P_s, but if the waves were to get to Marienburg at the observed time they would have to travel there from Heligoland at an average velocity of 5·15 km./sec. The time of the first arrival at Minsen shows that the average velocity over the first 54 km. must be less than 4·6 km./sec., and the waves then have to pass near Soltau where the average velocity is about 4·4 km./sec. This leaves only about 150 km. of path unaccounted for, over which the velocity would have to be well over 5·5 km./sec. Hypothesis (*c*) runs into difficulties in the reverse direction, but they turn out to be somewhat less severe. In this case the arrival at Marienburg is taken to be a late sample of P_g, and we can allow about 3 sec. for the time to pass down and up through the sediments. This leaves an average velocity of 5·45 km./sec. in the P_g layer, whilst the velocity over the first 120 km. is 5·95 km./sec. Thus the average velocity for the rest of the way would have to be about 5·0 km./sec., but the true velocity in the rock could be higher if the waves were to travel by an indirect path. (*b*) is not very consistently supported, and it would be surprising if waves along the postulated path could give rise to arrivals as strong as those at Göttingen and Jena, where the ordinary refracted wave seems to be totally absent. This hypothesis, however, has to be reintroduced in connexion with P_n, and must therefore be regarded as a possibility in the present case.

If we accept hypothesis (*c*) as the most probable one, it would appear that the practice of giving a single velocity for P_g is a rougher approximation than is usually believed. The mean velocity for all the onsets is about 5·1 km./sec. The wide departure from normality makes it difficult to assess the uncertainty, but a value of $\pm 0·3$ or 0·4 km./sec. would cover most of the range of the residuals. It appears from this that improvements in observatory technique are unlikely to increase the consistency of P_g times much beyond the present level, but that more attention might be devoted to the use of data from small groups of stations to map the local variations.

(*f*) *P waves from the lower layers*

The first arrivals at distances greater than 120 km. define a straight line with a velocity of more than 8·1 km./sec. At the shorter distances the curve is dominated by the results obtained along the Göttingen line. Kolding, which is the only station not on this line, gives a residual of $-0·83$ sec. This is more than four times the standard deviation and may be associated with the suggested rise of the P_g layer under southern Denmark. It was decided to omit this reading from the computation, and thus to obtain a line which would refer strictly to the north German plain. At the other end of the range, the Alpine station at Chur gives a residual of 0·64 sec., which can be explained on the very moderate assumption of an extra 3 km. of sediment. The remaining stations give a velocity of $8·18 \pm 0·16$ km./sec., with the standard deviation of one observation equal to 0·19 sec. This value is significantly higher than that given in the earlier tables (Jeffreys 1939), but agrees better with the Burton-on-Trent explosion and with the most recent work on European near earthquakes (Jeffreys 1947 *a*, *b*).

Of the other residuals, the largest are 0·42 sec. at Copenhagen and $-0·30$ sec. at Göttingen. The latter value agrees with the tendency of stations in central Germany to give slightly early readings of natural earthquakes, and the late arrival at Copenhagen may appear only as a result of the excessive weight given to the Göttingen region.

The second arrivals were first treated on the assumption that they represented a phase (corresponding to Conrad's P_x) travelling with a velocity of $7·72 \pm 0·19$ km./sec. The third arrivals gave a possible phase which might correspond with P^*. Examination of the residuals showed at once that P_x at Benthe would have to be rejected, and threw some doubt on the identification of the other arrivals at the stations before Copenhagen.

In order to test the data in this range of distance we compare the observed successes of the lines with the average density of arrivals between P_n and P_g. The values of s for P^* and P_x are respectively 0·6 and 1·7 sec., and the group of stations provides seven arrivals spread over an average time of about 8 sec. The observed successes for P^* and P_x are 2 and 1 respectively, whilst the numbers expected within the same distance from arbitrary lines would be 1 and 3. Recomputation of P_x without the intermediate stations gives a velocity of $8{\cdot}3 \pm 0{\cdot}2$ km./sec. and a starting time of $13{\cdot}9 \pm 1{\cdot}6$ sec. The velocity agrees with that of P_n, which suggests that the wave may travel for most of its time in the ultrabasic layer, being refracted to and reflected back from the free surface at one point in its path. The relation between the starting times is confirmed more accurately by the fact that the centroid of the P_x observations has a residual of $6{\cdot}95 \pm 0{\cdot}45$ sec. on P_n which agrees closely with the P_n intercept of $7{\cdot}37 \pm 0{\cdot}13$ sec. If the equation of P_x is deduced entirely from that of P_n, the standard deviation for one observation becomes 0·8 sec., and the number of successes in the whole line is 7. This is clearly a much better result than that of the trial line, which obtained only one success for a standard deviation of 1·7 sec.

Only seven observations remain between P_n and P_g, six of which might possibly be ascribed to P^*. The velocity derived from these is $6{\cdot}4 \pm 0{\cdot}16$ km./sec., but the line scores only two successes for a standard deviation of 1·6 sec. In general, the postulate of an additional layer should only be made when the evidence in its favour is strong, and when all the possible phases which could be produced by a simpler structure have been considered. The present explanation of P_x suggests that further readable phases might be produced by greater numbers of reflexions, and waves of this type have been described in short-range seismic work (Bullard, Harland, Gaskell & Kerr-Grant 1940). In view of this and other possibilities it is not considered that the existence of P^* is required by the Heligoland results. If, however, the existence of P^* is confirmed by other work in the area, the Heligoland data could be used to improve the determination of its travel times.

(g) *Depths of layers*

The constants which have been given for P_g and P_n are sufficient to determine the thickness of one layer below the rocks which transmit P_s. If an additional layer is postulated, the data of table 3 have to be divided between two phases, and are then insufficient to give a good determination of the constants. The additional data which are required could be drawn from the stations farther from Heligoland, or from near earthquake work. The latter source is superior for the present purpose. We therefore take velocities of 5·57 and 6·50 km./sec. for P_g and P^* and calculate the intercepts by extrapolating back from the appropriate groups of arrivals in table 3. The observations within 67 km. of the shot point fix the intercept of P_g at 1·65 sec., whilst the others give an intercept of 3·45 sec. for P^*. The depths according to the two hypotheses are given in table 4.

The observed range of velocity of P_s introduces an uncertainty of about ± 1 km. Local variations in the propagation velocity of the other phases will affect the thicknesses of individual layers, but the errors tend to compensate each other in the final summation. Trial solutions suggest that the total uncertainty in the depth of the ultrabasic layer is about ± 2 km. on either hypothesis.

TABLE 4

phase	velocity	thickness of layer (km.)	phase	velocity	thickness of layer (km.)
P_s	4·4	6·7	P_s	4·4	5·9
P_g	5·95	20·7	P_g	5·57	8·3
P_n	8·18	—	P^*	6·50	15·7
			P_n	8·18	—
	total	27·4		total	29·9

(*h*) *Later arrivals*

After the passage of P_g the motion becomes somewhat irregular, and is composed largely of waves of longer period. These were not recorded at all well by the short period vertical instruments, and the observations from the permanent stations are few in number. The difference between the amplitudes of P and S waves was smaller than is usual for near earthquakes. It was therefore difficult to identify the arrivals on the basis of appearance. The significance of lines drawn through groups of arrivals is therefore examined by the method of § 3 (*d*).

Twenty-nine arrivals which might be S or Rayleigh waves were recorded between Göttingen and Paris, spread over an average duration of 60 sec. This gives a mean density of nearly 0·5 arrival per sec., so that any line supported by the data must show significantly more than one success per second of standard deviation.

At Göttingen, a very strong arrival at 77·5 sec. was taken as S_n, and a line was fitted to it and the most plausible onsets at Copenhagen, Uccle, Collm and Stuttgart. This gave a velocity of 4·82 km./sec., but the high value was due mainly to a large negative residual at Stuttgart. On rejecting this point we find a satisfactory velocity of $4{\cdot}36 \pm 0{\cdot}06$ km./sec., but the starting time of $4{\cdot}3 \pm 1{\cdot}3$ sec. is before that of P_n. It is possible that such an effect might occur if S were generated by refraction from a primitive P movement, but as the velocity of S_n is less than that of P_g the appropriate critical angle is imaginary. The ray path is therefore not one of least time, and the resultant train of S waves would not be expected to have a sharp beginning. Strong arrivals at Göttingen and Kew, and weaker ones at Kolding, Cambridge, Stuttgart and Paris, may be Rayleigh waves. The velocity is $3{\cdot}01 \pm 0{\cdot}04$ km./sec. and the starting time $2{\cdot}6 \pm 1{\cdot}5$ sec.

S_n and R each score two successes, whilst the numbers expected to fall within the same range about lines drawn at random would be 0·4 and 1·6 respectively. The lines are not significant, therefore, except in so far as they help to explain the presence of strong arrivals, or agree with theoretical predictions. In spite of the crudity of the test, it does indicate that acceptable values of the standard deviations of parameters are not sufficient criteria of the reliability of assumed phases.

(*i*) *Unidentified features*

Tracings of the beginnings of records from the near stations (figure 6) show that some of the first arrivals are feeble, and are followed by strong, later onsets. At Spijk, the use of a line of geophones fixes the velocity of the second arrival at about 8 km./sec., and suggests that it might be P_n. The second arrivals at Esens, Altenwalde, Møgeltønder and Bentshöhe might be explained in the same way, but the residuals (table 1) are large. Moreover, the second arrival is not conspicuous at Westerholz or Jübeck. There is a strong second arrival at Wurfreihe which is too early for P_n. Minsen and Poppenbüll recorded a single shock. H.M.S. *Nepal* recorded the rectified output from a hydrophone in the water. The trace shows a complicated series of peaks, of which nothing beyond the first arrival has been explained.

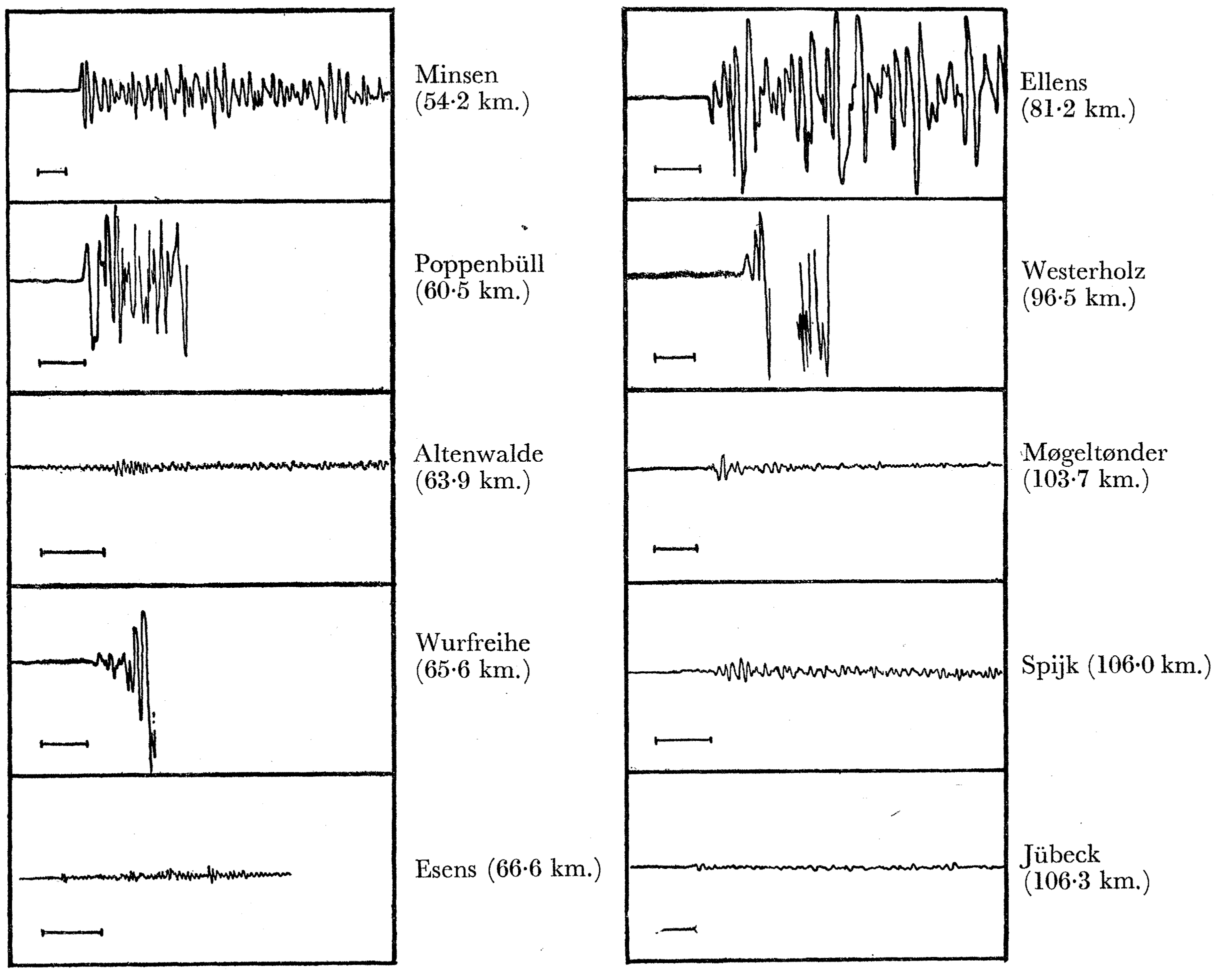

FIGURE 6. Beginnings of seismograms at near stations. The short straight lines show the distance moved by the paper in 1 sec.

These features provide further evidence that no simple layer structure will account for all the results, but quite small variations within the layers might produce multiple arrivals by providing alternative paths for the waves. If this is the correct explanation, the multiple arrivals should be included separately in the summations, but an attempt to do this consistently runs into insuperable difficulties in the identification of phases. The decision to rely on first arrivals was made for this reason, but may result in the chosen onsets having less than average transmission times. The high apparent velocity of P_g might arise partly from this cause.

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A number of other onsets were recorded by the near stations, at times up to a minute after the first shock. These may perhaps be due to waves transmitted through the upper sediments, but are too erratic to be interpreted in detail.

5. Amplitudes

(a) *The total energy*

The vertical displacements of the ground were determined at the seven stations for which the characteristics of the instruments were sufficiently well known. The energy density E is related to the amplitude a and frequency f of the motion by the equation

$$E = \tfrac{1}{2}(2\pi af)^2\rho,$$

where ρ is the density of the rock.

If a train of waves of amplitude a is reflected at a free surface, the displacement of the surface may be as much as $2a$. On the other hand, many of the waves considered were refracted up from deeper layers, and it is probable that the amplitude of these was substantially less than that of the waves which continued to travel downwards. The effect of these two factors is difficult to estimate, but it seems probable that substituting the amplitude of the surface displacement for a would give a fairly conservative estimate for the mean energy density. The few records from horizontal instruments showed that the horizontal displacements were at least as great as the vertical ones, so that the total energy is about three times that of the vertical component (i.e. $a = \sqrt{3} \times$ vertical component).

The total energy is obtained by multiplying E by the volume within which the energy density can be regarded as uniform. In most near earthquakes, the bulk of the energy appears to be carried by the long waves which follow S_g (Jeffreys 1947 *a*), and the energy of the Burton-on-Trent and Oppau explosions was calculated on the assumption that the disturbance extended to a depth of 18 km. In the present study, the relative amplitude of the early arrivals is far greater than usual, so that an assumption of hemispherical propagation may be nearer the truth. The volumes affected in the two cases are $2\pi D\Delta l$ and $2\pi\Delta^2 l$ respectively, where l is the length of the wave train, and D is the depth of penetration of cylindrical waves.

Table 5

station	distance (cm.) ($\times 10^7$)	a ($\times 10^{-5}$)	$2\pi f$	E $\times (10^{-6})$	l (cm.) ($\times 10^6$)	total energy: cylindrical propagation $D = 18$ km. ($\times 10^{16}$)	total energy: hemispherical propagation ($\times 10^{17}$)
Møgeltønder	1·03	20·2	37	73	4	3·3	1·9
Kolding	1·77	13·0	43	41	10	8·1	8·0
Copenhagen	3·33	5·2	12·5	0·55	20	0·41	0·76
Stuttgart	6·08	1·6	6·3	0·013	60	0·039	0·13
Strasbourg	6·22	3·0	6·3	0·046	48	0·16	0·54
Paris	7·04	1·6	6·3	0·013	80	0·085	0·33
Puy de Dôme	9·98	1·3	9·8	0·021	96	0·22	1·2

The estimates of energy vary by a factor of 60 on the hemispherical assumption, and by 200 on the cylindrical one. The estimate for the Burton-on-Trent explosion was 5×10^{16} ergs on the cylindrical assumption and was based on a single station at a distance of 123 km. The Oppau explosion and the Hereford earthquake gave about the same value. In the Heligoland explosion, the amplitudes were much smaller and the frequencies higher than in the other examples. This difference may be largely instrumental, and if the Heligoland instruments missed the long waves whilst the others missed the short ones, the estimates of energy may be too small in all cases.

(*b*) *Effect of tamping*

The total thermal energy available in the Heligoland charge was 13×10^{19} ergs. This is almost the same as at Burton-on-Trent and about twice that at Oppau. The Oppau explosion was on the surface of the ground whilst the others were buried, but the difference in efficiency seems to have been small. We therefore consider what effect was to be expected from the top cover at Heligoland.

Suppose that, if the explosion had been unconstrained, the kinetic energy in the diverging gas would have been W ergs. If the mass of explosive was m grams and the velocity of the gases was v, we would have $W = \frac{1}{2}mv^2$, and the total momentum would be given by

$$p = K\sqrt{(2mW)},$$

where K is a constant of order 1, depending on the distribution of momentum in various directions.

If the gases had given up energy W' to a tamping mass M, the total momentum would be given by

$$p' = K_1\sqrt{\{2m(W-W')\}} + K_2\sqrt{(2MW')},$$

where the numerical constants K_1 and K_2 take account of the energy lost by turbulence, and of the new distribution of momentum with direction. It will be assumed that neither of these constants will be much greater than K.

At Heligoland, the main crater must have extended almost to sea-level, although the bottom is now filled with about 15 m. of rubble. The original plateau was about 50 m. above sea-level, and the diameter of the crater is about 100 m. Thus the volume is about $1{\cdot}3 \times 10^{11}$ cm.^{3}, making $M = 3 \times 10^{11}$ g.

An upper limit to the velocity imparted to the rock can be obtained from the fact that nearly all of it fell within 100 m. of the centre of the crater, and the lower limit from the fact that a large amount was able to clear the rim. Taking 50 m. as an estimate of the average height attained gives $W' = 1{\cdot}5 \times 10^{18}$ ergs.

The total thermal energy in the main charge group was about $1{\cdot}3 \times 10^{20}$ ergs, or roughly 100 times that required to produce the crater. Under the most favourable circumstances W would be about 30 % of the thermal energy. The mass m is equal to that of the explosive, together with the mass of the air which is accelerated during burning. The air within a radius of 60 m. would weigh as much as the explosive, so that the contribution from this source would become important if the burning time were to exceed a few hundredths of a second.

Thus we have $$W'/W \doteqdot 1/30, \quad M/m < 100,$$

giving $$p' < K_1\sqrt{(2m \times 29/30W)} + K_2\sqrt{(2 \times 100m \times W/30)} \doteqdot 2K\sqrt{(2mW)}.$$

Hence the presence of the tamping cannot increase the momentum in the ground by a factor of more than 2 or 3.

(c) *Appearance of onsets*

Pronounced differences were observed between the sharpness of the onsets at different stations. The effects at the near stations have already been discussed (§ 4 (i)), and a further example can be drawn from the P_g arrivals at Bokel (figure 7). The two records were obtained on a pair of similar instruments about 400 m. apart. The station was resting on a salt-dome.

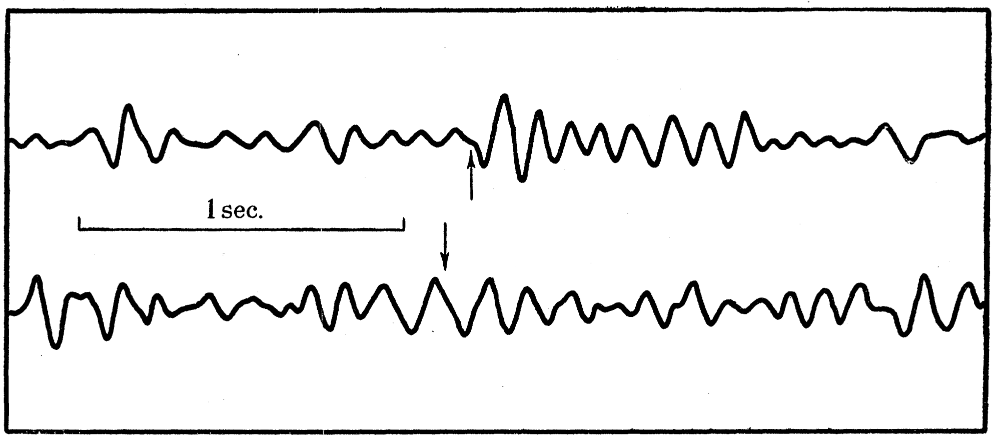

FIGURE 7. P_g onsets at two similar seismographs near Bokel. The instruments were separated by about 400 m. The separation of the arrows indicates the expected interval between corresponding arrivals on the two records.

On the basis of the ray theory of propagation, the energy density in a wave front diverging from a point on the surface of a horizontally stratified medium is given by the expression

$$E = A\frac{\cot e}{\Delta}\left|\frac{de}{d\Delta}\right|,$$

where e is the angle of emergence and A is a constant for the phase. When a wave is refracted out of a plane layer in which it is travelling parallel to the surface, e is the complement of the critical angle, and the expression vanishes for all values of Δ. It might then be expected that the surface disturbance would be produced entirely by the energy which is diffracted out of the rectilinear paths, and would be lacking in wave-lengths short compared with the dimensions of the geological structure. More detailed considerations, based on the theory of wave propagation (Muskat 1933), show that the refracted wave does have an appreciable amplitude, even if the layers are plane and uniform. Nevertheless, it appears that the energy reaching a point on the surface may be substantially increased by any irregularities which would refract or scatter the waves across the boundary at angles other than the critical one. If this argument is correct, we should regard emergent arrivals as typical of a uniform structure, whilst impulsive ones might indicate the presence of irregularities. This suggestion is supported by an observation from the Witwatersrand, where a station resting on a granite batholith gives consistently better records than several neighbouring ones. It has been confirmed that the effect is a property of the site, rather than of any particular seismograph (Gane, Hales & Oliver 1946).

(*d*) *Relative amplitudes of P_n and P_x*

In spite of the loss of energy in the double refraction, there are several stations at which P_x is more strongly received than P_n. This may be due in part to the fact that P_x is continually fed with energy from the P_n waves reflected from the surface and therefore tends to grow at the expense of the latter phase.

Moreover, strong reception of P_n at a given station depends on conditions being favourable for refraction at a particular point in the lower layer, whilst it is possible for P_x to be formed from the wave refracted at a point nearer the focus, and reflected in the vicinity of the station. P_x therefore has a double chance of reaching a particular seismograph, and may therefore be relatively immune from the erratic variations of amplitude which affect other phases. At greater distances, waves involving large numbers of reflexions might become prominent.

The identification of P_x as a reflected phase is important, in so far as it simplifies the structure which has to be postulated to explain the results. It is possible that some of the observations which have been ascribed to P^* might arise from the same cause.

6. Comparison with other studies

Parts of the data treated in the present paper have been covered by reports from Förtsch, Reich & Schulze (preliminary reports from Göttingen), by Rothé (1947) and by Charlier (1947). Rothé deduced velocities of 8·2 and 5·5 km./sec. for P_n and P_g at the French stations. The other results are summarized in table 6.

Table 6

profile	velocities and nomenclature		
	Göttingen reports	Charlier	Willmore
Soltau: N.W.	6·82 ‘Permian or below’	—	6·65 ± 0·07 P_g
S.W.	6·07 ‘Permian or below’	—	5·60 ± 0·06 P_g
W.	5·45 ‘Permian or below’	—	6·3 ± 0·5 P_g
E.	5·85 ‘Permian or below’	—	6·0 ± 0·8 P_g
Heligoland: Oldenburg	6·2 P^*	—	6·0 ± 0·2 P_g (or P^*)
Schleswig	6·8 inclined P^*	—	6·78 ± 0·03 P_g (or P^*)
mean of near stations	—	—	5·95 ± 0·14 P_g
Göttingen	8·3 P_n	—	—
Heligoland, distant stations	8·1 P_n	7·973 P_n	8·18 ± 0·014 P_n
	—	7·572 X_1	8·18 P_x
	6·4 P^*	6·573 P^*	6·4 ± 0·16 P^*?
	5·1 P_g	5·481 $\bar{P}$	variable P_g
	4·3 S_n	4·483 S_n	4·36 ± 0·06 S_n?
	—	4·150 X_3	—
	3·7 S^*	3·773 S^*	—
	2·9 $\bar{S}$	3·229 $\bar{S}$	3·01 ± 0·04 R?

The observations used to determine the velocities at the near stations were almost the same for the Göttingen workers and the present author, except that the results were used in Göttingen without the sedimentary corrections. As the stations were chosen to make these corrections as uniform as possible this difference usually had little effect on the velocities, and the main discrepancies arise from the fact that my values were based entirely on first arrivals. The most significant difference is in the velocity along the Soltau west profile, where the Germans included two late arrivals at short distances and rejected a first arrival at 25·6 km. As the velocity was based on only five observations spread over a range of 10 km., it was very sensitive to changes of this type. The difference along the south-west profile is produced entirely by the sedimentary corrections.

Charlier's lower value for P_n appears to have arisen from the chance selection of observations, as does the difference between his X_1 and my P_x. His P^* was based on three arrivals, of which the one at Uccle seems to have been extremely feeble. The P^* observations quoted by Göttingen were essentially the same as those discussed in the present paper.

The velocities given for P_n are higher than those given in the 'Near Earthquake Tables' (Jeffreys 1939), but agree more closely with the most recent work of Jeffreys (1947 *a*, *b*) and with the earlier studies of Gräfe & Conrad. Velocities of more than 8 km./sec. have been given for P_n in North America, but the values for Central Asia and Japan appear to be somewhat lower.

7. Notes on apparatus and technique

In view of the possibility of further experiments of this type, it is considered that a summary of the features to be expected would not be out of place.

(*a*) *Characteristics of arrivals*

It is probable that the dominant frequencies of the waves will not be greatly affected by conditions at the source, provided that the duration of the shock is short in comparison with the period of the arrivals. The Heligoland explosion probably satisfied this condition with respect to the waves observed at distances exceeding 100 km., so that the figures in table 5 should have a fairly general application. The frequencies observed in the Soltau explosions were usually less than 10 c./sec., but some of the higher frequencies here may have been absorbed by the loose material under the shot point.

(*b*) *Instruments*

The small Schweydar and Wiechert seismographs (magnification 20,000, period $\frac{1}{5}$ sec.) were suitable for recording the early arrivals up to about 200 km., but probably missed information about the longer waves. At greater distances the best records came from seismographs with a good response between 1 and 10 c./sec., and a fairly sharp cut-off at lower frequencies. It was possible to use magnifications at which the microseisms were strongly recorded, as the frequency difference was sufficient to enable weak arrivals to be detected through a much larger background amplitude. A greater number of instruments recording the horizontal motion would have assisted in the identification of phases, and more uniformity between stations would have been desirable.

For recording at sea, quartz hydrophones resting on the bottom, or buoyed at a depth of at least $\frac{1}{8}$ of a wave-length below the surface, are suitable (Hill & Willmore 1947). H.M.S. *Nepal* recorded a peak of 1200 dynes/sq.cm. for P_g at a distance of 52 km. from Heligoland. The background disturbance was about 20 dynes/sq.cm.

The recording speed should be sufficient to enable the traces to be read to within a few hundredths of a second. A speed of 1 cm./sec. was found to be the easiest to read with the naked eye, and at this speed a magnification of 60,000 could be used. Higher magnifications might prove useful under quiet conditions.

(c) *Arrangement of stations*

In short range refraction work, it is usual to arrange stations along straight lines, because the travel-time curve for a plane layer will be linear. By shooting the same line of stations in two directions, it is possible to determine the true velocity and the dip separately.

The object of arranging the work in this way is to concentrate attention on the smallest possible sample of the structure, so that repeated experiments will yield precise information. The method is less satisfactory if only a single focus is available, and it may be more informative to spread the stations as evenly as possible over the country. When two foci can be used it may be worth while to run a line of stations between them, and it is always desirable for some of the stations to be common to both shocks. If the advantages of this had been realized in time, the author would have recommended recording the Heligoland tremors by at least one station on each of the Soltau profiles, and by a station at the Soltau shot point. Much of the ambiguity would then have been removed from the results of both experiments.

The explosion at Heligoland was primarily a military operation, and it was only possible to obtain so much seismological information through the willing co-operation of the Naval authorities. In particular, Commander Ward and Lt-Com. Graves, R.N., who were in charge of the Heligoland operations, cheerfully submitted to a considerable complication of their plans to assist the work. The Soltau explosions were arranged by the Explosives Storage and Transport Committee of the War Office, and the author would like to express his gratitude to them and to Lt-Col. Finney for their assistance. The arrangements for the installation and operation of the field stations were made possible by the Research Branch of the Control Commission, and particularly through the personal efforts of Dr R. Frazer and Mr H. L. P. Jolly. The Admiralty provided H.M.S. *Nepal* and a ship in the mouth of the Forth, whilst the German Hydrographic section and North German Fisheries Control provided more observers and permitted the use of the trawler *Astrid*. Radio facilities were provided by the B.B.C., B.F.N. and the German network.

It must also be acknowledged that this paper is a discussion of results largely obtained by others. My thanks are due to all those who arranged for the operation of temporary stations, particularly Dr Veldkamp in Holland, Miss Lehmann in Denmark, and to Professor Bartels, Professor Reich, Dr Förtsch and Dr Schulze who arranged for so many and such successful observations in Germany. I also had the advantage of reading the preliminary reports by Professor Rothé, Dr Charlier and the Göttingen workers, and of discussions with those who attended the Royal Society meeting on the Heligoland Explosion.

Finally, I must acknowledge the continued assistance and encouragement of Professor E. C. Bullard, who was largely responsible for the original conception of the experiment. I am deeply indebted to him, and to Professor H. Jeffreys and Professor O. T. Jones, for helpful advice and criticism of the manuscript.

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